Essential Topics for Secondary Mathematics Success: What Mathematics Teachers Think

Melinda Evans
Monash University
<melinda.evans@monash.edu>

In this preliminary study, to inform a larger study where Year 8 students create an online module for peers, I surveyed mathematics teachers (n = 30) on essential mathematics topics: (a) most critical for students' success, (b) most conceptually challenging for students, and (c) in which more fluency is needed, as well as (d) their likelihood of considering an online course as an intervention. Fractions concepts, times tables, and equation solving were most critical for success; students need more understanding of fractions concepts, and more fluency in both fractions concepts and times tables. Online course use addressed teachers' concerns for students in essential mathematics topics.

In this paper, I outline a preliminary questionnaire-based study conducted on what topics mathematics teachers rate as most important and needed for students to succeed in their study of mathematics at secondary school. The results are needed to help inform the focus topic for a larger design-based intervention study in which Year 8 students create an online or e-learning module for peers. In this novel approach, Year 8 students change roles from content consumers to content creators. Learning outcomes and engagement levels will be tracked during the creation process. Also, if mathematics teachers are concerned about their students' understanding or fluency in essential mathematics topics, the questionnaire asked the likelihood of teachers considering an online course or module, as I aim to produce in the major study, to address those concerns.

Background

Disengagement from mathematics by lower secondary students is widespread in Western nations (e.g., Middleton, 2013). In Australia, Martin, Anderson, Bobis, Way, and Vellar (2012) found that disengagement from mathematics in middle years students (Years 6 to 8; n = 1,601) was correlated with the following student and classroom factors: low mathematics self-efficacy, low valuing of mathematics, reduced enjoyment and perceived classroom enjoyment, mathematics anxiety, and perceived classroom disengagement. Balfanz, Herzog, and Mac Iver (2007) noted that course failure in US lower secondary schools "dramatically dampens a young adolescent's perceived control and engagement" (p. 224). Furthermore, systemic secondary school practices in mathematics education may not be meeting the following young adolescents' needs that were mostly better met in primary school: a degree of autonomy, social interaction and relatedness, a close relationship with their teachers, small group work, challenging activities that require higher order thinking (Eccles et al., 1993), and the motivation of a hard, specific group goal (Locke & Latham, 2006). Giving students fresh opportunities to succeed while attention is drawn away from the individual performance and towards a challenging, relevant goal could assist students with concentrating on learning mathematics and not on any previous negative experiences with the subject. To this end, a novel approach has been devised such that a class of Year 8 students works together to produce an e-learning module for peers.

However, the topic on which the inaugural study is created needs to be selected carefully. Ideally, to enhance the project's relevance, value, and wider appeal, the topic

(2017). In A. Downton, S. Livy, & J. Hall (Eds.), *40 years on: We are still learning! Proceedings of the 40th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 237-244). Melbourne: MERGA.

needs to be one in which other more complex topics depend and is demonstrably essential and therefore valuable to school mathematics, further education, civic life, and the workplace. Enough students need to have some degree of difficulty with the topic such that the finished product, an e-learning module for local peer use, will be seen as a potentially worthwhile and challenging project to work on and a meaningful resource for end-users. In order to willingly commit to the research project, mathematics teachers also need to be able to appraise the topic as worthy of expending effort, time, and resources.

Previous research supports that intervention on the topic of fractions is needed as it is often poorly understood across a broad spectrum of learners: primary school students (Daraganova & Ainley, 2012; Zhang, Clements, & Ellerton, 2015), middle years (Years 5 to 8) students (Clarke & Roche, 2009; Stafylidou & Vosniadou, 2004), more senior high school students (Brown & Quinn, 2006; Kloosterman, 2010), and the general public (Basic Skills Agency, 1997; Reyna & Brainerd, 2008). The Longitudinal Study of Australian Children, Annual Statistical Report 2011 (Daraganova & Ainley, 2012) included primary school teachers' ratings (n = 3,533) of children's numeracy skills (aged 8 to 9 years) and found that a quarter of children had either not yet (7%) or were just beginning (16%) to form an age-appropriate concept of fractions compared to, for example, half that amount either not yet (3%) or just beginning (9%) to form an age-appropriate concept of place value. In an open questionnaire asking Australian middle years students themselves (Years 5 to 8; n = 3562) about their single most important aspiration in mathematics, increased understanding of fractions, decimals, and percentages was the highest response (Wilkie, 2016). However, missing from the literature and to better support the most needed topic in mathematics for middle years students is the standpoint of the mathematics teachers, which this preliminary study aims to help address.

There are other essential mathematics topics that are apposite contenders on which to base a novel intervention study in Year 8 mathematics. Referring to Martin et al. (2012), students need to see the value or relevance of mathematics in order to best engage in the subject. The following essential number and algebra topics are in the Australian Curriculum (AC; Australian Curriculum, Assessment and Reporting Authority, 2016), thereby are relevant to school-based education, and are mentioned in or inferred from the Programme for the International Assessment of Adult Competencies, Australia (Australian Bureau of Statistics, 2013), thereby are relevant to civic and workplace needs: mental computation; multiplication facts (or times tables); estimating; negative numbers; place value of decimals; computing with decimals; percentage of a quantity; percentage change; converting between decimals, fractions, and percentages; repeating patterns; growing patterns; order of operations; and solving equations. Also, the topic of fractions is quite broad and could be split into fractions concepts and computing with fractions.

While it is generally agreed that fluency in multiplication facts is an essential aim of primary education (e.g., Wong & Evans, 2007), it is not clear what importance secondary mathematics teachers hold the automaticity and flexible use of multiplication or times tables facts, and which particular group of facts is important for students to learn or needs extra attention. A recent search in the U.S. database, Educational Resources Information Center (ERIC), of peer-reviewed publications using the Boolean term "and" with the ERIC subjects "multiplication", "computation", and "mathematics instruction" revealed 77 articles, but none of these were research studies or discussion on which multiplication facts students need to be fluent in to recognize factors, support derived strategies, appreciate patterns (like the repeated digit pattern: 11, 22, 33…, of the 11 times table), or calculate commonly encountered multiple quantities quickly, like the number of months in multiple

years. Researched support for the benefits of fluently learning multiplication facts only up to 10 x 10, as currently required in the AC, versus learning up to the 12 times tables of yesteryear was not found. An aim of this study is to survey what multiplication facts mathematics teachers deem as necessary for students to fluently learn.

A further aim of this preliminary study is to gauge the likelihood of mathematics teachers using an online or e-learning course — in the form of set of lessons/modules inclusive of competency-based assessment — to address students' deficits in understanding that prevent progress and success. In previous decades, this question would be invalid because the most consistent reason for mathematics teachers not using technology in their classrooms was lack of adequate access to computers (Forgasz, 2006; Zammit, 1992). Now, however, the computer to student ratio in Victorian government schools is nearly one-to-one (1:1.46) in primary schools and better than one-to-one (1:0.94) in secondary schools (Department of Education and Training, 2016). It appears likely that students would engage with a digital resource. Young Australians are avid users of technology, with 99% of 15- to 17-year-olds in 2014-2015 having access to the internet and an average of 18 hours per week use (Australian Bureau of Statistics, 2016). In Adelaide, Paris (2004) found that secondary students usually preferred online supplements to their classroom learning compared to pen and paper-based tasks.

Online courses have multiple advantages for users. They allow for immediate feedback (Butler, Pyzdrowski, Goodykoontz, & Walker, 2008) and easy, global, and rapid connection to other resources (e.g., the digital learning objects repository, Scootle, by Education Services Australia, 2017). Online resources offer asynchronous (anytime) use, mobility, anonymity, and they can be text-based, use multimedia, or be multimodal (Haythornthwaite & Andrews, 2011) and be potentially accessed by unlimited numbers of students. Pertinent to the main study here, digitally composed courses are highly editable and can be quickly and relatively cheaply published either locally or globally.

Use of online courses in U.S. public high schools is widespread, especially for the purposes of regaining credit for failed courses and completing core requirements in the main academic subjects (Clements, Stafford, Pazzaglia, & Jacobs, 2015). In Sydney region secondary schools, Neyland (2011) found a range of attitudes to online courses by computer coordinators, from aversion to sheer dedication. While there is some research on Australian mathematics teachers' beliefs affecting their choices to use technology in general (e.g., Hennessy, Ruthven, & Brindley, 2005; Pierce & Ball, 2009), there is little on their willingness to use online or e-learning courses or modules as interventions where students need extra help in essential topics to progress and succeed in their studies.

There are five research questions for this study:

- Which topics do mathematics teachers rate as critical for success in secondary mathematics?
- In which topics do students need more conceptual understanding?
- In which topics do students need more fluency?
- What multiplication facts do students need to learn?
- What is the likelihood of mathematics teachers offering an online course to their students to increase their understanding of mathematics topics required for success in secondary mathematics?

Method and Data Analysis

A questionnaire was conducted at the 2015 annual Mathematics Association of Victoria conference in Melbourne. Teachers of mathematics were approached and asked to complete a brief questionnaire about the most important topics required for secondary students' success in mathematics. The survey was anonymous, but respondents were asked to indicate the year levels in mathematics they had taught in the last five years. Space was provided for participants to record any further thoughts. Respondents were approached at morning tea break. Teachers were asked to tick the top three mathematics topics in three columns:

1. Topics critical for mathematics success in secondary mathematics
2. Topics in which students need more conceptual understanding
3. Topics in which students need more fluency

The following topics, each with sub-topics in italics, were included as choices: mental computation (including using known facts flexibly, time tables - with a grid to select any or all from 2 to 12, and estimating), negative numbers (computing with negative numbers), fractions (including two subtopics of understanding all fractions concepts and computing with fractions), decimals (including place value of decimals; and computing with decimals), percentage (including percentage of a quantity, percentage change, and converting between decimals, fractions, and percentages), algebra (including repeating patterns, growing patterns, BODMAS [i.e., order of operations – brackets, orders, division, multiplication, addition, and subtraction], and solving equations), and other.

Despite asking participants to tick the top three topics in each column, some ticked more and some less. Three respondents selected more than three topics per column. In these cases, so that no one participant's scores dominated the results, the total score of three was divided evenly across each of that respondent's responses for that column. For example, one participant ticked six subtopics (Only the top three were requested per column), and as such each selected subtopic was assigned 3/6 = 0.5 points. One respondent selected fewer than three subtopics for one column and his or her score was not altered.

A further section asked respondents the following question to rate with yes, no, or maybe: If an online course or module was available to help address the above concerns you have for your students, would you be most likely to consider: using it in your classroom, setting it as homework, or mentioning it as a resource that students can follow up in their own time?

Indicating a concern with the questionnaire, the two subtopics, mental computation and times tables, were quite often ticked ambiguously for the first column, which prompted for the topics critical for secondary mathematics success (6 out of 22 respondents who selected these two topics did not respond clearly, for example, ticking mental computation, but not times tables, then selecting "all" for times tables) and in those cases, each of these two sub-topics was assigned a score of 0.5. This was not an issue for the conceptual understanding and fluency prompting columns that were marked unambiguously for both times tables and mental computation. Reported tallied scores which included averaged data were rounded to the nearest whole number to better reflect the precision of that data.

Results

Thirty completed surveys were collected. All teachers surveyed had taught either upper primary mathematics (Years 5 and 6) or secondary mathematics (Years 7 to 12), with the greatest majority teaching from Year 9 through to senior secondary (Years 11 and 12)

mathematics. One secondary teacher had retired more than five years prior to the survey whereas all others had been teaching in the last five years. No respondents had taught mathematics at preschool or at a technical and further education (TAFE) college. One respondent had written "VCE equivalent" for year level of mathematics taught and his or her data were included under "General mathematics".

The mathematics levels taught by number of respondents in the last five years were as follows: Early years to Year 4 (2), Years 5 and 6 (3), Year 7 (9), Year 8 (11), Year 9 (15), Year 10 (18), Foundation or Essential mathematics (4), Further mathematics (16), General mathematics (16), Mathematics methods (17), Specialist mathematics (7), university-level mathematics (7), Technical and Further Education (TAFE) mathematics (0), and online (1).

In answer to the prompt, "I think the following three maths topics… are critical for success in secondary maths: (Tick your top 3)", the highest-scoring topics understanding all fractions concepts (14/30), times tables (13/30), and solving equations (13/30). The clear choice in which the 30 surveyed teachers thought students need more conceptual understanding was fractions concepts. It was selected 16 times as one of the top three. Other choices selected with about half the frequency of the top choice were computing with negative numbers, place value of decimals, growing patterns, and solving equations. The mathematics topic choices for which the 29 surveyed teachers (One respondent did not complete this section) thought students need more fluency were fractions concepts (chosen 13 times) and times tables (chosen 11 times). Computing with fractions and solving equations were the next most frequent choices, as depicted in Figure 1.

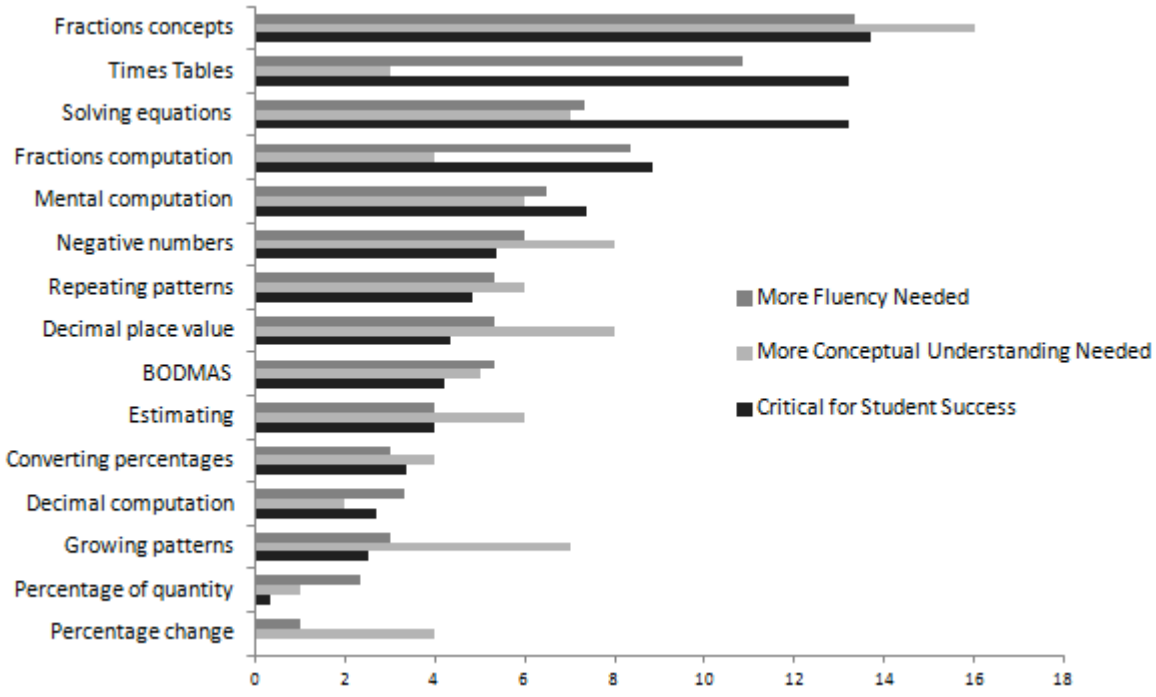

Figure 1. Mathematics teachers' appraisals (n = 30) of essential mathematics topics most critical for success, and in which there are student deficits in conceptual understanding or fluency.

Where times tables were selected across all three categories – more fluency needed, more conceptual understanding needed, or critical for student success – the majority of selections for times tables was "all" (24 out of 33 total selections), meaning here the multiplication factors from 2 to 12. However, in about one-fifth of instances, the selection of which particular times tables required was simply omitted, and one respondent selected factors from 2 to 10 and another indicated that only the seven and eight times tables required further improvements in fluency for students.

Most respondents selected that they would consider using an online course in their classroom to support students with difficulties that they had identified in essential mathematics (18 selected yes, 4 selected no, and 5 selected maybe). Most respondents would also consider setting an online course for homework (yes: 16, no: 4, and maybe: 4) and as a resource for students to pursue in their own time (yes: 16, no: 2, and maybe: 3).

Discussion and Conclusion

The questionnaire results successfully confirmed that fractions concepts are the best choice for the focus intervention topic for the larger study involving the co-creation of an online module by Year 8 students for peers. Fractions concepts were selected as the most frequent choice by secondary and middle years mathematics teachers across all three categories – critical for success in secondary mathematics, more conceptual understanding is required, and more fluency is required. The findings support earlier research (Basic Skills Agency, 1997; Brown & Quinn, 2006; Clarke & Roche, 2009; Kloosterman, 2010; Reyna & Brainerd, 2008; Stafylidou & Vosniadou, 2004; Zhang et al., 2015) that showed that fractions concepts are poorly understood by many students and the general public. Furthermore, the middle years and secondary mathematics teachers' perspectives found here align with that of primary teachers' appraisals (Daraganova & Ainley, 2012) and that of middle years students themselves (Wilkie, 2016) that fractions concepts are often the most problematic for students.

Despite the small number of participants (n = 30), the non-random participant selection, and the questionnaire layout initially prompting ambiguity in the responses between times tables and using known facts flexibly (mental computation), there is, albeit qualified, support for students increasing their fluency in times tables in general and learning multiplication facts with factors up to 12. Most (73%) selections for times tables were "all", meaning here the multiplication factors from 2 to 12. However, the questionnaire did not include the multiplication facts for zero and one, and did not allow for easy, explicit choice between students knowing just up to the single-digit factors (zero through nine), from 0 to 10, the products up to 100 (e.g., 0 x 99 = 0, 3 x 3 = 9, 5 x 15 = 75, 48 x 2 = 96), a concentration on prime number factors and deriving the rest, or any other possibility or combination. Nonetheless, especially in the dearth of information and research on this topic in the literature, it warrants further exploration as to what secondary mathematics teachers regard as the most important multiplication facts for students to be taught, learn, understand, practice, recall, and be able to fluently use, and why.

There was strong support for considering the use of an online course to address concerns that respondents had for their students in essential mathematics topics. This augurs well for finding support from mathematics teachers for the creation or use of an online course in their classrooms and ties in with Paris' (2004) finding that the students themselves prefer web-based rather than pen and paper-based supplements. However, there is only scant research on the use of online or e-learning courses as interventions in Australian high schools. This raises the following questions: What online intervention

courses are being used? What criteria do teachers use when selecting an online course for students? For what topics do teachers seek online courses for their students?

If further research with a wider participant base replicates the findings here, that both fractions concepts and multiplication fact fluency are not only vital for secondary mathematics success but also merit remedial intervention, then perhaps a broader-level intervention for improving fraction fluency and conceptual knowledge and multiplication fact fluency in lower secondary school, or earlier, is warranted. Online interventions have the advantage of inexpensively and quickly being made available to unlimited numbers of recipients as long as a central repository or other means for dissemination for such resources is available. Alternatively, the students themselves could, with assistance, create local resources that they need to succeed in their mathematics education to share with peers, and in the process, improve in their engagement with the subject.

References

Australian Bureau of Statistics. (2013). *Programme for the International Assessment of Adult Competencies, Australia, 2011-12 (Report No. 4228.0)*. Retrieved from http://www.abs.gov.au/ausstats/abs@.nsf/Latestproducts/4228.0Main%20Features12011-12

Australian Bureau of Statistics. (2016). *Household use of information technology, Australia, 2014-15. (Report No. 8146.0)*. Retrieved from http://www.abs.gov.au/ausstats/abs@.nsf/mf/8146.0.

Australian Curriculum, Assessment and Reporting Authority. (2016). *Australian Curriculum, v8.3 F-10 Curriculum: General capabilities, Numeracy*. Retrieved from http://www.australiancurriculum.edu.au/generalcapabilities/numeracy/introduction/introduction.

Balfanz, R., Herzog, L., & Mac Iver, D. J. (2007). Preventing student disengagement and keeping students on the graduation path in urban middle-grades schools: Early identification and effective interventions. *Educational Psychologist, 42*(4), 223-235.

Basic Skills Agency. (1997). *International numeracy survey: A comparison of the basic numeracy skills of adults 16-60 in seven countries*. Retrieved from http://search.proquest.com/docview/62591759?accountid=12528

Brown, G., & Quinn, R. J. (2006). Algebra students' difficulty with fractions: An error analysis. *Australian Mathematics Teacher, 62*(4), 28-40.

Butler, M., Pyzdrowski, L., Goodykoontz, A., & Walker, V. (2008). The effects of feedback on online quizzes. *International Journal for Technology in Mathematics Education, 15*(4), 131-136.

Clarke, D. M., & Roche, A. (2009). Students' fraction comparison strategies as a window into robust understanding and possible pointers for instruction. *Educational Studies in Mathematics, 72*(1), 127-138. doi:10.1007/s10649-009-9198-9

Clements, M., Stafford, E., Pazzaglia, A. M., & Jacobs, P. (2015). *Online course use in Iowa and Wisconsin public high schools: The results of two statewide surveys.* Washington, DC: Regional Educational Laboratory Midwest.

Daraganova, G., & Ainley, J. (2012). Children's numeracy skills. *The longitudinal study of Australian children: Annual statistical report 2011.* Melbourne: Australian Institute of Family Studies.

Department of Education and Training. (2016). *Census of computers in schools*. Retrieved from http://www.education.vic.gov.au/school/principals/infrastructure/Pages/censuscomputers.aspx

Eccles, J. S., Midgley, C., Wigfield, A., Buchanan, C. M., Reuman, D., Flanagan, C., & Mac Iver, D. (1993). Development during adolescence: The impact of stage-environment fit on young adolescents' experiences in schools and in families. *American Psychologist, 48*(2), 90-101.

Education Services Australia. (2017). *Scootle.* Retrieved from https://www.scootle.edu.au/ec/p/home

Forgasz, H. (2006). Factors that encourage or inhibit computer use for secondary mathematics teaching. *Journal of Computers in Mathematics and Science Teaching, 25*(1), 77-93.

Haythornthwaite, C. A., & Andrews, R. A. (2011). *E-learning theory and practice*. Los Angeles, CA: SAGE.

Hennessy, S., Ruthven, K., & Brindley, S. (2005). Teacher perspectives on integrating ICT into subject teaching: Commitment, constraints, caution, and change. *Journal of Curriculum Studies, 37*(2), 155-192. doi:10.1080/0022027032000276961

Kloosterman, P. (2010). Mathematics skills of 17-year-olds in the United States: 1978 to 2004. *Journal for Research in Mathematics Education, 41*(1), 20-51.

Locke, E. A., & Latham, G. P. (2006). New directions in goal-setting theory. *Current Directions in Psychological Science, 15*(5), 265-268. doi:10.1111/j.1467-8721.2006.00449.x

Martin, A. J., Anderson, J., Bobis, J., Way, J., & Vellar, R. (2012). Switching on and switching off in mathematics: An ecological study of future intent and disengagement among middle school students. *Journal of Educational Psychology, 104*(1), 1-18.

Middleton, J. A. (2013). More than motivation: The combined effects of critical motivational variables on middle school mathematics achievement. *Middle Grades Research Journal, 8*(1), 77-95.

Neyland, E. (2011). Integrating online learning in nsw secondary schools: Three schools' perspectives on ICT adoption. *Australasian Journal of Educational Technology, 27*(1), 152-173.

Paris, P. G. (2004). E-learning: A study on secondary students' attitudes towards online web assisted learning. *International Education Journal, 5*(1), 98-112.

Pierce, R., & Ball, L. (2009). Perceptions that may affect teachers' intention to use technology in secondary mathematics classes. *Educational Studies in Mathematics, 71*(3), 299-317. doi:10.1007/s10649-008-9177-6

Reyna, V. F., & Brainerd, C. J. (2008). Numeracy, ratio bias, and denominator neglect in judgments of risk and probability. *Learning and Individual Differences, 18*(1), 89-107.

Stafylidou, S., & Vosniadou, S. (2004). The development of students' understanding of the numerical value of fractions. *Learning and Instruction, 14*(5), 503-518.

Wilkie, K. J. (2016). *Exploring middle school girls' and boys' aspirations for their mathematics learning.* Paper presented at the 40th Annual Conference of the International Group for the Psychology of Mathematics Education, Praha, Czech Republic.

Wong, M., & Evans, D. (2007). Improving basic multiplication fact recall for primary school students. *Mathematics Education Research Journal, 19*(1), 89-106.

Zammit, S. A. (1992). Factors facilitating or hindering the use of computers in schools. *Educational Research, 34*(1), 57-66.

Zhang, X., Clements, A. M., & Ellerton, N. F. (2015). Conceptual mis(understandings) of fractions: From area models to multiple embodiments. *Mathematics Education Research Journal, 27*(2), 233-261.